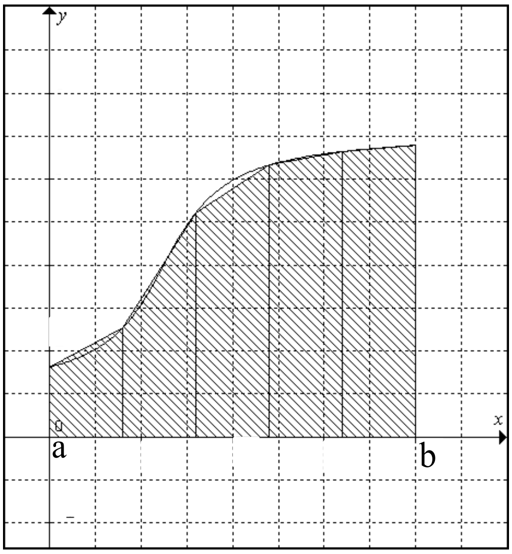


Recall that the area of a trapezoid is $A = \frac{1}{2} h (b_1 + b_2)$

Height of each trapezoid: $h = \frac{b-a}{n}$

Lengths of bases of ith trapezoid:

$f(x_i)$ and $f(x_{i-1})$ where $x_0 = a$ and $x_n = b$.

So, $\int_a^b f(x)dx$

$$= \frac{1}{2} \cdot \frac{b-a}{n} \left[[f(x_0)+f(x_1)]+[f(x_1)+f(x_2)]+ [f(x_2)+f(x_3)]+ [f(x_3)+f(x_4)]+[f(x_4)+f(x_5)] \right]$$

The Trapezoidal Rule

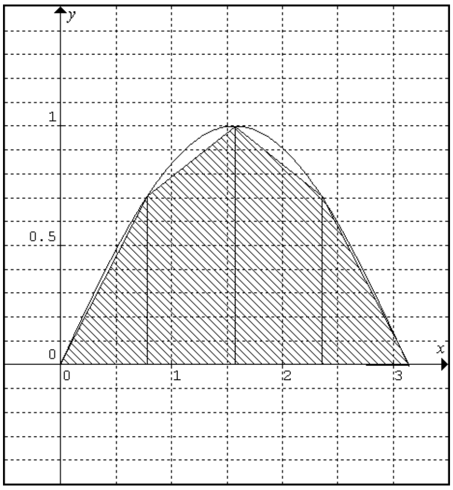
$$\int_a^b f(x)dx = \frac{b-a}{2n}[f(x_0)+2f(x_1)+ 2f(x_2)+2f(x_3)+...+2f(x_{n-1})+f(x_n)]$$

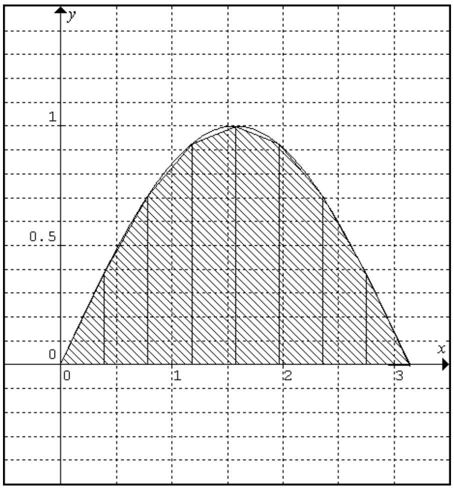
where $a=x_0$ and $b=x_n$

4.6 Numerical Integration

First Method: The Trapezoidal Rule

ex. Approximate $\int_0^{\pi} \sin x \, dx$ using n=4 and again with n=8.





ex. Estimate $\int_1^2 \frac{1}{x}\,dx$
using the Trapezoidal Rule with n=4.

Error:

1. The error in approximating $\int_a^b f(x)dx$
 using the Trapezoidal Rule is: $E \leq \frac{(b-a)^3}{12n^2}[\max |f''(x)|]$
 for x in [a,b].